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fixed by us, or are, or are supposed infinitesimal.

This indeed ought opportunely to be noted in remaining subsequent propositions.

Proposition IV. *But inversely (the figure of the preceding proposition remaining) the angles at the join CD will be right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB.*

Proof. For if the straight CD is equal to the opposite AB, and nevertheless the angles at it are either obtuse, or acute; now these such angles prove it (P. III.) not equal, but less, or greater that the opposite AB; which is absurd against the hypothesis.

The same uniformly avails in regard to the remaining cases. It holds therefore that the angles at the join CD are either right, or obtuse, or acute, according as the straight CD is equal, or less, or greater than the opposite AB. Quod erat demonstrandum.

Definitions. Since (P. I.) the straight joining the extremities of equal perpendiculars standing upon the same straight (which we call base), makes equal angles with these perpendiculars; therefore there are three hypotheses to be distinguished about the species of these angles. And the first indeed I will call hypothesis of right angle; the second however, and the third I will call hypothesis of obtuse angle, and hypothesis of acute angle.

ARITHMETIC.

Conducted by B.F.FINKEL,Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

16. Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.

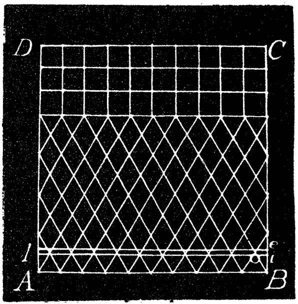
How many stakes can be driven down upon a space 15 feet square allowing no two to be nearer each other than 1½ feet, and how many allowing no two to be nearer than 1¼ feet?

Solution by B. F. FINKEL, A. M., Professor of Mathematics, Kidder Institute, Kidder, Missouri.

(*a*) 1. Since the least distance from one stake to another is 1½ ft., the number of 1½ ft. spaces in the base line AB is 15 ft.÷1½ft. or 10. Hence, we can place 11 stakes on the base line AB, and, by *square arrangement*, we can place on the square $ABCD$ 11 rows with 11 stakes in a row, in all 11×11 stakes or 121 stakes.

2. By *quincunx arrangement*, we can place 11 stakes on the base line AB and over these, as vertices of equilateral triangles 10 stakes. Now the width Bi of the strip $ABil$ is $\sqrt{oB^2-oi^2}=\sqrt{(1\frac{1}{2})^2-(\frac{3}{4})^2}=\frac{3}{4}\sqrt{3}$ ft. $=1.2990381+$ ft. Hence, the width of the strip le is 1.5 ft. $-1.2990381+$ ft. $=2009619-$ ft.

Therefore, for every strip $1\frac{1}{2}$ ft. wide, there is a gain, by the *quincunx arrangement*, of .2009619 ft. Now if we gain .2009619 ft. for 1 strip $1\frac{1}{2}$ ft. wide, how many $1\frac{1}{2}$ ft. strips must be taken to gain $1\frac{1}{2}$ ft.? By Proportion, we have .2009619 ft. : $1\frac{1}{2}$ ft. :: 1 strip $1\frac{1}{2}$ ft. wide : (?=7 strips $1\frac{1}{2}$ ft. wide). Hence, in 7 strips $1\frac{1}{2}$ ft wide, we have 8 strips 1.2990381 ft. wide, and therefore have 9 rows of stakes, 5 rows of 11 in a row and 4 rows of 10 in a row—in all, 95 stakes.



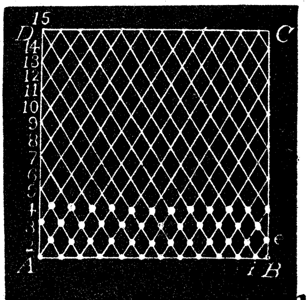
It is seen from the figure that there are 3 strips left. On these three strips we can make 3 rows of 11 stakes in a row, or 33 stakes on the remaining strips using the *square arrangement*. Hence, we can place 95 stakes +33 stakes or 128 stakes on the square.

(*b*) 1. When the least distance from one stake to another is $1\frac{1}{4}$ ft., the number of $1\frac{1}{4}$ ft. spaces on the base line AB is 15 ft. ÷ $1\frac{1}{4}$ ft. or 12. Hence, we can place 13 stakes on the base line AB, and, by *square arrangement*, we can place on the square 13 rows of 13 stakes in each row—in all, 13×13 stakes or 169 stakes.

2. By *quincunx arrangement*, we can place 13 stakes on the base line and over these, as vertices of equilateral triangles, 12 stakes.

Following the same method as (2) of (*a*), we find we can have 5 rows of 13 stakes in each row or 65 stakes and 4 rows of 12 stakes in each row or 48 stakes—in all 113 stakes. Now there will be 5 strips left, on which, by *square arrangement*, we can have 5 rows of 13 stakes in each row or 65 stakes. Hence, by the *square and quincunx* arrangements combined, we can place 113 stakes +65 stakes or 178 stakes on the square.

3. If we could have 15 rows of 12 stakes in each row, instead of 14 rows, 10 rows having 13 stakes each and 4 rows having 12 stakes each, we would have 180 stakes. If such an arrangement is possible, we must place the stakes in the rows more than $1\frac{1}{4}$ ft. apart and thus allow the rows to come closer together. Since we are to have 15 rows, there will be 14 strips, each 15 ft. ÷ 14 or $\frac{15}{14}$ ft. wide. By placing the first row of 12 stakes on the base line AB (Fig. 2), we have $11\frac{1}{2}$ spaces (the least number of spaces) each 15 ft. ÷ $11\frac{1}{2}$ or $\frac{30}{23}$ ft. In Fig. 2, $Bi=\frac{1}{2}$ of $\frac{30}{23}$ ft. $=\frac{15}{23}$ ft. and $Be=\frac{15}{14}$ ft. $\therefore$ $ei=\sqrt{Bi^2+Be^2}=\sqrt{(\frac{15}{23})^2+(\frac{15}{14})^2}$ $=15\sqrt{\frac{14^2+23^2}{23^2\times14^2}}=\frac{75}{322}\sqrt{29}=1.2512+$ ft. Since this is more than the allowable distance, 1.25 ft., the arrangement is possible and we can place 180 stakes on the square, by arranging them as shown in Fig. 2.



This problem was also solved by *H. W. HOLYCROSS, G. B. M. ZERR, M. A. GRUBER, H. C. WHITAKER*, and *I. L. BEVERAGE*.

Mr. Holycross and Mr. Beverage get the same answers as above, but Mr. Beverage says that the 180 stakes are placed in a manner similar to the 128 stakes. This is not true. The other results differ from the above and from each other.